INTEGER RULES

+ To Add Integers

- Same Signs (++ or - -): Add absolute values. Use the same sign. absolute value.
  - 4 + 7 = 11
  - -3 + (-6) = -9
- Different Signs (+ - or - +): Subtract absolute values. Use the sign of the larger
  - 4 + (-7) = -3
  - -3 + 10 = 7

- To Subtract Integers

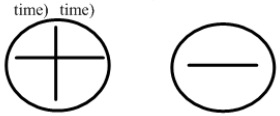
NO! L.C.O. Leave Change Opposite

Follow Addition Rules

- 4 - (-6) = becomes 4 + (+6) = 10

x ÷ To Multiply or Divide Integers

- Same Signs (++ or - -): The answer is always positive (2 numbers at a time) time)
  - (-2) x (-7) = 14
  - 15 ÷ 3 = 5
- Different Signs (+ - or - +): The answer is always negative (2 numbers at a
  - (-2) x 5 = -10
  - 8 ÷ (-4) = -2



VOCABULARY

Absolute Value (Distance from Zero) - always expressed as a positive shown as bars around a number $|-5| = 5$ and $|5| = 5$

Additive Inverse (Opposite) - same absolute value but with the alternate sign when additive inverses are added the result is always zero.

Fraction Decimal %

(Just a selfie of a number.)

$\frac{1}{5}$ Divide (top number goes in the box) .2 Move decimal 2 places right 20%

$\frac{41}{1000}$ Read it Reduce it Don't use "point" .041 Move decimal 2 places left 4.1%

DECIMALS

5.928 ? 3.8

+ Add / Subtract

Line up the decimal.

$$\begin{array}{r} 5.928 \\ +\ 3.8 \\ \hline 9.728 \end{array} \qquad \begin{array}{r} 5.928 \\ -\ 3.8 \\ \hline 2.128 \end{array}$$

x Multiply

Count the number of digits behind the decimal. Move it over that many places left in the PRODUCT.

$$\begin{array}{r} 5.928 \\ \times\ 3.8 \\ \hline 22.5264 \end{array}$$

÷ Divide

Outside - to the end.
Inside - same number of places.
Straight up into answer.

$$3.8\overline{)5.928} \quad 1.56$$

POWERS

a way of writing repeated multiplication

7^3 = 7 x 7 x 7 = 343

Base: 7 Exponent: 3

$8\frac{3}{4} \ ? \ 2\frac{4}{5}$

What are you doing?

Add or Subtract → Common Denominator

If you change the bottom you must change the top.

*Don't go to Improper Fraction

Add	Subtract
Once in common denominator, add or subtract the tops and leave the bottoms the same.	If you need to borrow, you borrow the number that is in the denominator.
$8\frac{15}{20} + 2\frac{16}{20} = 10\frac{31}{20}$	$7\frac{15+20=35}{20} - 2\frac{16}{20} = 5\frac{19}{20}$
Simplify $11\frac{11}{20}$	Simplify $5\frac{19}{20}$

Multiply or Divide → Improper Fraction

Horseshoe Method

*Don't go to Common Denominator

Multiply	Divide
You can CROSS SIMPLIFY!!!!!! $\frac{\cancel{35}^{7}}{\cancel{4}_{2}} \times \frac{\cancel{7}^{7}\cancel{14}}{1\cancel{5}} = \frac{49}{2}$	NO! Flip the 2nd fraction to its reciprocal. Then change the problem to multiplication. $\frac{\cancel{35}^{5}}{\cancel{4}} \times \frac{5}{\cancel{14}_{2}} = \frac{25}{8}$
Simplify $24\frac{1}{2}$	Simplify $3\frac{1}{8}$

ORDER of OPERATIONS

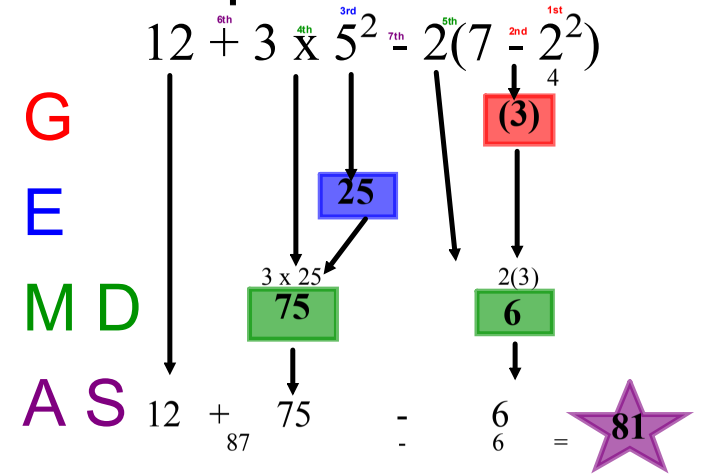
G Grouping Symbols - Parenthesis, Fraction Bars, Absolute Value

E Exponents and roots

M D Multiplication and Division-Left to Right
**M is the same level as D
so...DM is the same as MD
DO THESE LEFT TO RIGHT

A S Addition and Subtraction- Left to Right
**A is the same level as S
so...AS is the same as SA
DO THESE LEFT TO RIGHT

Order of Operations EXAMPLE



Combing Like Terms

$4f + 5 + 2m + 13 + 9m - 2f + 4 + 3f^2 + m^2$

Terms - the parts of an expression that are added together

$\boxed{4f} + \boxed{5} + \boxed{2m} + \boxed{13} + \boxed{9m} + \boxed{-2f} + \boxed{4} + \boxed{3f^2} + \boxed{m^2}$

Coefficient - the number part of a variable term Understood

$\boxed{4}f + 5 + \boxed{2}m + 13 + \boxed{9}m + \boxed{-2}f + 4 + 3f^2 + \boxed{1}m^2$

Constant terms - a terms with no variable(just numbers)

$4f + \boxed{5} + 2m + \boxed{13} + 9m - 2f + \boxed{4} + 3f^2 + m^2$

Like terms - terms that have the same variable part to the same power

Like	Like	Like	Like	Like
$4f$	5	$2m$	$3f^2$	$1m^2$
$-2f$	13	$9m$		
	4			

Simplified - $2f + 22 + 11m + 3f^2 + m^2$

Descending Order- $3f^2 + m^2 + 2f + 11m + 22$

Properties of Operations

Commutative Property (Addition and Multiplication)
Order DOES NOT matter in addition and multiplication (Travel)
Examples: a + b = b + a 5 x 3 = 3 x 5 5 + (-2) = -2 + 5

Associative Property (Addition and Multiplication)
The grouping DOES NOT matter in addition and multiplication (Grouping can Change)
Examples: 5 + (15 + 8) = (5 + 15) + 8 4(5 x 9) = (4 x 5)9

Distributive Property
Everybody in the (group) gets some. (Everybody in the group gets some)
Examples: 3(5p + 4) = 15p + 12 7(2 - 9y) = 7(2) - 7(9y)

Identity of Addition
Anything plus zero is ITSELF.
Examples: y + 0 = y -4 + 0 = -4 26 + 0 = 26

Identity of Multiplication
Anything times ONE is ITSELF.
Examples: 1g = g 55 x 1 = 55 -7 x 1 = -7

(DNA)

Inverse of Addition
Anything plus its opposite (additive inverse) is 0 (Undo to get back to zero)
Examples: -8 + 8 = 0 h + -h = 0 18 + -18 = 0

Inverse of Multiplication
Anything times its reciprocal (multiplicative inverse) is 1 (Undo to get back to one)
Examples: $k \times \frac{1}{k} = 1$ $5\frac{1}{3} \times \frac{3}{16} = 1$ $\frac{2}{9} \times \frac{9}{2} = 1$

Zero Property
Anything times ZERO is ZERO (Zero of anything is zero)
Examples: w x 0 = 0 -6 x 0 = 0 210 x 0 = 0

Solving 2 Step Algebraic Equations

U-Turn Method

$9 + 4b = -11$

1. Find the variable.
2. Identify what has been done to the variable. (Pay special attention to the order in which it was DONE.)
3. Undo it in reverse order.

This is what was already done to b.

b → x 4 → + 9 → -11

This is how we UNDO what had been done.

-5 ← ÷ 4 ← - 9 ← -11

-11 - 9 = -20 then -20 ÷ 4 = -5

$9 + 4(-5) = -11$

Check your answer by substituting it back into the original equation to ensure the answer makes the statement true.

9 + (-20) = -11

-11 = -11

Cancellation Method

$9 + 4b = -11$

$-9 \quad = -9$

$4b = -20$

$\div 4 \quad = \div 4$

$b = -5$

1. Since the b was 1st multiplied by 4 and then 9 was added to it...
2. We will UNDO what was done in the opposite order...
3. By subtracting 9 from both sides...
4. and then dividing both sides by 4.

Solving Algebraic Equations

$3(f+1) = 5f - 25 + 2f$

1. **Leave Change Opposite** (only the subtraction)

$3(f+1) = 5f + (-25) + 2f$

2. **Distributive Property** (break them out of jail)

$3f + 3 = 5f + -25 + 2f$

3. **Combine Like Terms** (get your pennies with your pennies and your nickels with your nickels)

$3f + 3 = 7f + (-25)$

4. **If variables are on both sides... subtract one set from both sides.**

$3f + 3 - 3f = 7f + (-25) - 3f$

5. **You are in a 2-Step Algebraic Equation**

$3 = 4f + (-25)$

6. **Solve with U-Turn or Cancellation**

U-turn Method

3 = 4f - 25

f → x 4 → - 25 → 3

7 ← ÷ 4 ← +25 ← 28

7 = f

Cancellation Method

$3 + 25 = 4f + (-25) + 25$

$28 = 4f$

$28 \div 4 = 4f \div 4$

$7 = f$

7. **Substitute answer into the original equation to check it.**

$3((7) + 1) = 5(7) - 25 + 2(7)$

$24 = 24$

This is true so 7 is the solution.

Inequalities

$<$ Less Than

$\leq$ Less Than Equal To (*At most)

or

$>$ Greater Than

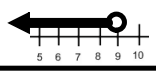
$\geq$ Greater Than or Equal To (*At least)

Inequality are solved in the same way algebraic equations are solved...with 2 exceptions.

1. The answer will be a range of possible solutions.
2. If you multiply or divide by a negative number (while solving) you need to flip the inequality sign.

Compare

Equation	Inequality
9 more than twice a number is 12 less than 27	9 more than twice a number is less than 27
	*Notice that we do not know HOW much less than 27 it is. This means we will have a range of solutions.
$2n + 9 = 27 - 12$	$2n + 9 < 27$
$2n + 9 = 15$	$2n < 18$
$2n = 6$	$n < 9$
$n = 3$	Exception 1 Range of solutions: n is less than 9 so... 8.895, 6, -12, -15.7, etc.



Exception 2

When you multiply or divide by a negative number you flip the inequality sign.

$7 - 5p \leq -3$

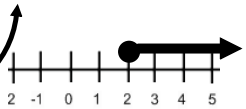
$7 + -5p \leq -3$

$-5p \leq -10$

$p \geq 2$

If the opposite of 5p is $\leq$ THEN 5p will be $\geq$.

Think about it this way... If you would have divided by 5 rather than -5 you would have ended with $-p \leq -2$. If the opposite of p is $\leq$ then p must $\geq$.

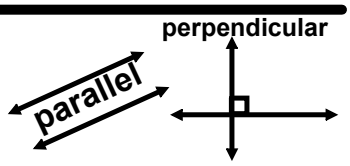


Geometry

Triangle

sum of the interior angles is 180°

Area = $\frac{bh}{2}$



Circumference = πd

Area = $r^2\pi$

$\pi \approx 3.14$

Quadrilaterals

- sum of the interior angles is 360°

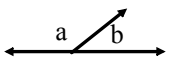
-**Parallelogram** - 2 pairs of parallel sides which are also congruent (opposite angles are congruent)

1. Rhombus – 4 congruent sides
2. Rectangle – 4 right angles (4 congruent angles)
3. Square – 4 congruent sides and 4 right angles

Area = bh

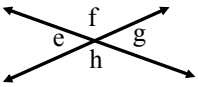
-**Trapezoid** - 1 pair of parallel sides, CAN NOT have all congruent sides or angles

Area = $\frac{1}{2}h(b_1 + b_2)$



∠a and ∠b are **supplementary** (add to 180°)

∠c and ∠d are **complementary** (add to 90°)



∠e and ∠g and also ∠f and ∠h are **vertical angles** (opposite angles) and thus **congruent**

3-dimensional solid with parallel and congruent Bases

Volume = Bh

*Capital B stands for area of the base

Ratios, Rates, C of P

A RATIO compares two numbers.

1. ORDER MATTERS
2. There are 3 ways to write a ratio.
   1. as a fraction (3/4) 2. the word to (3 to 4) 3. using a colon (3:4)
3. The /, :, and to are read as "to"
4. Can be simplified (just like a fraction-but not to mixed number
5. MIGHT be a fraction BUT MIGHT NOT.

Rates - ratio of two quantities measured in different units (different labels)

1. Must have labels
2. Can be made into a UNIT RATE (Constant of Proportionality) by writingan equivalent rate with 1 as the denominator

   370 miles to 8 hours

   $\frac{372 \text{ miles}}{8 \text{ hours}} = \frac{46.5 \text{ miles}}{1 \text{ hour}}$
3. Unit rates are generally read using the word PER.

Constant of Proportionality

is the UNIT RATE

x	3	4	9
y	18	24	54

Simplified $\frac{18}{3} = \frac{24}{4} = \frac{54}{9} = \frac{6}{1}$

formula is $\frac{y}{x}$

is the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ of a proportional relationship $\frac{6}{1}$ represents up 6 right 1 or down 6 left 1

is the coefficient of x in a proportional relationship so... $y = 6x$

is the y when x is 1 or $(1, \text{Constant of Proportionality})$ (1, 6)

Proportions

1. ORDER MATTERS
2. Cross Products are equal.
3. Are read a is to b as c is to d. $\frac{a}{b} = \frac{c}{d}$
4. Ratio = Ratio
5. You CAN NOT...CAN NOT...cross simplify proportions.
6. Have special connections.
7. USE LABELS when setting them up!

Proportions - Setting them up and Solving

In 17 hours of working Mr. Dellenbach earned $156.74. How long will it take him to earn $750?

To set up a proportion

1. write a ratio you know
2. use your labels
3. make sure you keep your connections because ORDER MATTERS

$\frac{\$156.74}{17 \text{ hrs}} = \frac{\$750}{? \text{ hrs}}$

If I went $ to hrs on the first ratio...I go $ to hrs on the second ratio.

To solve a proportion

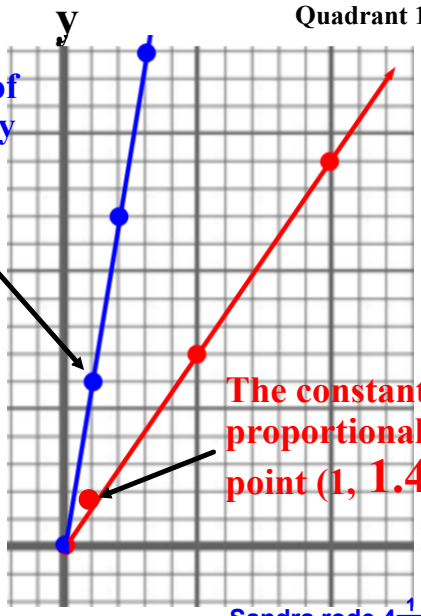
1. KNOW that cross-products are equal
2. multiply the two numbers that are diagonal from each other
3. then divide this product by the remaining number (the number diagonal from the unknown number)

$\frac{\$156.74}{17 \text{ hrs}} = \frac{\$750}{? \text{ hrs}}$

since cross-products are equal...

$17 \times 750 = 156.74 \times h$

$\frac{17 \times 750}{\$156.74} \approx 81.3 \text{ hours}$



The constant of proportionality point (1, 6).

The constant of proportionality point (1, 1.40).

James rode 5 rides for $7 at the fair. What was the unit rate?

$\frac{\$7}{5 \text{ rides}}$

Sandra rode $4\frac{1}{2}$ miles in $\frac{3}{4}$ of an hour. How fast was she riding?

$\frac{4\frac{1}{2} \text{ miles}}{\frac{3}{4} \text{ hour}}$

1. Put y over x. If there is not a specific y or x, read the labels as a rate to determine which makes better sense.
2. Divide to find the unit rate/constant of proportionality. This will be your slope. Sometimes it is easier to leave it improper (such as 7/5)

$1.40 per ride better as... $\frac{7}{5}$

6 mph

3. Every proportional line goes through the origin (0,0). Use the slope (directions up/down then left/right to graph the proportional line.

From (0,0) up 7 right 5.

From (0,0) up 6 right an understood 1.

4. You can then write the equation for the relationship in the form y = mx where m is the unit rate/constant of proportionality.

$y = \frac{7}{5}x$ or $y = 1.4x$

$y = 6x$

Remember to plot a point (,) airplane (,).

To use a slope...you go from a point

Percent

$\frac{\text{is}}{\text{of}} = \frac{\%}{100}$ or $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$

- a ratio with a denominator of 100.
- compares to a whole.
- means divide by 100
- is like a common denominator.
- is just a selfie of a number
- must be changed to a decimal or fraction in order to multiply or divide with it.

Percent Change/Percent Error

$\frac{\text{amount of change/error (the increase, decrease, or error)}}{\text{original amount}} = \frac{\% \text{ change}}{100}$

Properties of Operations

1. **Commutative Property** (+ and x) "Travel - Switch order"

   $5 + 3 = 3 + 5$ $b + 9 = 9 + b$ $7 \times 4 = 4 \times 7$ $-3 \times 2 = 2 \times -3$

2. **Associative Property** (+ and x) "Grouping changes"

   $(2 + 4) + 9 = 2 + (4 + 9)$ $(3 \times 5) \times 4 = 3 \times (5 \times 4)$

3. **Distributive Property** "Everyone in the group gets some"
   "Break it out of jail"

   $4(3p + 5) = 4x3p + 4x5$ or $12p + 20$ $5(12 - 3) = 60 - 15$

4. **Identity Property** "It is what it is. That is its DNA."

   Anything + 0 = itself $5 + 0 = 5$ $m + 0 = m$

   Anything x 1 = itself $12 \times 1 = 12$ $g \times 1 = g$

5. **Zero Property** "Zero it out"

   Anything x 0 = 0 $9 \times 0 = 0$ $d \times 0 = 0$

6. **Inverse Property** "Cancel it out"

   Anything + its additive inverse (opposite) = 0 $8 + -8 = 0$

   Anything x its multiplicative inverse (reciprocal) = 1

   $\frac{3}{7} \times \frac{7}{3} = 1$ $4\frac{1}{8} \times \frac{8}{33} = 1$ $4 \times \frac{1}{4} = 1$